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PRELIMINARY ANALYSIS OF HOT SPOT FACTORS IN AN ADVANCED REACTOR FOR SPACE ELECTRIC POWER SYSTEMS

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ABSTRACT

The maximum fuel pin temperature for nominal operation in an advanced power reactor is 1370 K. Because of possible nitrogen embrittlement of the clad, the fuel temperature was limited to 1622 K. Assuming simultaneous occurrence of the most adverse conditions a "deterministic" analysis gave a maximum fuel temperature of 1610 K. A statistical analysis, using a synthesized estimate of the standard deviation for the highest fuel pin temperature, showed probabilities of 0.015 of that pin exceeding the temperature limit by the distribution free Chebyshev inequality and virtually nil assuming a normal distribution. The latter assumption gives a 1463 K maximum temperature at 3 standard deviations, the usually assumed cutoff. Further, the distribution and standard deviation of the fuel-clad gap are the most significant contributions to the uncertainty in the fuel temperature.

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SUMMARY

Hot spot factors have been calculated for the fuel pins in a lithium cooled Advanced Space Power Reactor. These hot spot factors were estimated by calculating maximum temperatures in the uranium mononitride fuel. They account for uncertainties in power, coolant flow, material properties, and fuel pin dimensions. The hot spot factor is the ratio of the difference between the maximum fuel temperature and the coolant inlet temperature, with the uncertainties included, to the temperature difference for the nominal design condition.

The maximum fuel pin temperature for nominal design conditions is 1370 K (2006° F). A fuel temperature limit of 1622 K (2460° F) was set to prevent possible nitrogen embrittlement of the clad. This is a preliminary and, supposedly, conservative limit.

A "deterministic" analysis indicated a maximum temperature only 12 K (21.6° F) below an assumed fuel temperature limit of 1622 K (2460° F). This marginal safety factor under the assumption of simultaneous occurrences of the most adverse conditions indicates that a statistical analysis is desirable to obtain a more representative assessment of reliability.

A less pessimistic estimate of the maximum fuel pin temperature is 1463 K (2173° F) arrived at through a statistical analysis which assumes a normal distribution of the fuel temperature and a cutoff at three standard deviations. The probability of exceeding this temperature is, then, 0.0013. Using another statistical approach, an estimate was made of the probability of exceeding the fuel temperature limit based on the Chebyshev inequality which does not assume any particular mathematical function for the frequency distribution of the fuel temperature. This probability could be as much as 0.015 for the central pins.

The results of this study indicate the probability distribution function of the fuel-clad gap to be the most significant contributor to the uncertainty in fuel temperature.

INTRODUCTION

The performance of a reactor fuel pin is determined in part by the temperatures occurring within it. It is desirable to know the distribution of

temperature not only for design conditions, but also for any deviations from design which may possibly appear in an actual fabricated fuel pin functioning in a reactor core. Such information can be used to help guide the design by dictating materials, shapes, and dimensions, and setting requirements for tolerances. For instance the analysis can be used to establish which tolerances must be close and which may be relaxed from the point of view of fuel temperature effects.

This report deals with off-design effects on the maximum fuel pin temperature in an Advanced Power Reactor (APR) for Space Electric Power Systems investigated at the Lewis Research Center. It is desirable to know how much the temperature of a given spot may be influenced by off-design conditions. In particular, it is desirable to know what the maximum temperature in the fuel pin may be. Deviations from their design values may occur in a number of factors influencing temperature, and several such deviations may occur in the same pin, at the same place in the pin, and at the same time. However, it is not likely that all factors will have maximum deviations at the same location and at the same time.

Two approaches are used to estimate the maximum fuel pin temperature in the off-design condition. One approach, which is very conservative, assumes that all the variables are at their extreme expected values coincidentally and that each operates on the fuel pin temperature to produce its maximum value. This is the deterministic method which in its general form amounts to a worst case analysis. Amendola (ref. 1) formulated a deterministic method as a product of individual hot channel factors approximated by an abbreviated Taylor series expansion.

The second, more realistic, approach assumes the variables to be random statistical quantities. It is possible then to determine the frequency distribution of the fuel pin temperature by a method described by Abernathy (ref. 2). An alternative to that employs the assumption of a linear combination of independent variables but this time to obtain an approximation of the standard deviation of the fuel pin temperature. The standard deviation is needed to calculate the probability of exceeding a fuel temperature limit and to calculate the hot spot factor. The standard deviations of the independent variables are based on tolerances and published data since there is not yet any data on sample variations for this particular design.

In this report, the hot spot factor is calculated using both deterministic and statistical approaches. The hot spot factor is the ratio of the temperature difference between maximum fuel temperature and inlet coolant temperature for the off-design conditions and the temperature difference for the reference conditions, respectively. One technique (ref. 3) for quoting the hot spot factor by a statistical method is to assume that the fuel temperature for the off-design conditions is a temperature such that is has a probability of only 0.13 percent of being exceeded. This corresponds to three standard deviations of the fuel

temperature. However, as will be discussed later, the preliminary nature of the design dictates that a conservative estimate of probability of exceeding a fuel temperature limit be made and for this the Chebyshev inequality will be employed. Both the normal distribution approximation and Chebyshev inequality are probability estimation methods but the former assumes a particular distribution of the fuel pin temperature while the latter makes no assumption of a particular form of the frequency distribution.

This report includes a description of the fuel pin design, a discussion of the various uncertainties in variables affecting temperature, and a summary of the analytical techniques employed. The assumptions of independence and linearity of the variables are discussed briefly. The design radial and axial temperature distributions are discussed in Appendix B. A typical calculation of the hot spot factor is given in Appendix C.

DESCRIPTION OF REACTOR AND FUEL PIN

The reactor being studied has a fast spectrum and is fueled with uranium mononitride. An isometric drawing of the reactor is shown in figure 1. Each of 181 fuel pins is placed within a T-111 honeycomb structure to maintain a triangular pitch. The honeycomb structure is surrounded by an annular neutron reflector of TZM (Mo-0.5Ti-0.08Zr). The entire structure is surrounded by a T-111 pressure vessel which is 58.5 centimeters (23 in.) in diameter and 68.6 centimeters (27 in.) long. Reactivity is controlled by rotating six TZM drums in the annular reflector. Eleven fuel pins are placed in holes drilled on one side of each drum and an annular segment of the drum opposite the fuel pins contains a T-111 neutron absorber.

A sketch of a typical fuel pin and coolant channel are shown in figure 2. The UN fuel is clad with 0.147 centimeter (0.058 in.) thick T-111. A 0.0127 centimeter (0.005 in.) thick tungsten liner physically separates the fuel from the T-111 clad. This liner inhibits any chemical reaction between the UN fuel and tantalum in the clad. A clearance provided between the fuel and liner for ease of assembly, is filled with helium. During initial operation this nominal radial gap width is about 0.0038 cm. and closes as the fuel swells. Fission product gas collection spaces are provided at both ends of the pin and at the center of the fuel (the fuel cylinder is hollow). Fuel spacers are located at both ends of the fuel column to allow fuel swelling in the axial direction and to inhibit vibration of the fuel during launch conditions. Support buttons, located 120 degrees apart and at five axial positions on the honeycomb support tube, center the fuel element within the coolant channel and inhibit bowing.

The reactor design power is 2.17MW and the axial power shape can be approximated by a chopped cosine curve having a peak to average power ratio of 1.23. The radial power distribution across the reactor is flattened to 1.33 by using 3 fuel zones. The desired fuel loading in each of

the three zones is obtained by adjusting the diameter of the central hole in the fuel (table I). Pins containing the most fuel are located furthest from the center of the reactor. The greatest power density occurs in the center pin at the reactor midplane.

The fuel is cooled by liquid lithium which enters the reactor at 1165 K (1640° F). The primary lithium flow is in an annular space between the fuel and the honecomb. The average coolant temperature rise across the reactor is 55 K (100° F). The convective film coefficient is large so the clad wall temperature is nearly equal to the coolant temperature. The core design radial and axial temperature distributions are discussed in Appendix B.

DISCUSSION OF UNCERTAINTIES

Ten variables which contribute to the value of the highest fuel temperature in the reactor have been considered. The variation of each in the following description is quoted at two standard deviations except for the variables that are strictly a result of tolerance limits. These exceptions are local fuel thickness, orifice mismatch, flow annulus width, fuel-clad gap, fuel shift in the clad, and clad shift in the flow annulus. For these, the number of standard deviations associated with the half-range H is $\sqrt{3}$ or 1.732.

The entire list of variables is as follows:

(1) Overall Power Adjustment. Reactor power may be determined from measurements of fluid flow rate and coolant temperature difference from inlet to outlet. Variations from the desired reactor power can result from inaccuracy in the measurement of flow and temperature and time lags in the response pattern of the controls.

An uncertainty of ± 8 percent on overall power adjustment was assumed (see ref. 3). Although this variation (± 8 percent) is associated with three standard deviations in reference 9, in this study the ± 8 percent uncertainty is associated with only two standard deviations for conservatism.

(2) Local Power Density. The power density at a point in the fuel depends on the fuel density, enrichment, relative neutron flux, and neutron energy spectrum at that point. The fuel density and enrichment depend on the fuel fabrication methods while the relative neutron flux and spectrum depend on the nuclear properties and geometry of the materials in the reactor. An uncertainty of ± 4 percent in local power density is assumed.

(3) Local Fuel Thickness. The estimated uncertainty in fuel thickness is ± 0.0038 centimeter (± 0.0015 in.). This uncertainty is based on fuel fabrication tolerances.

(4) Local Thermal Conductivity of the Fuel. The thermal conductivity of the fuel depends on the fuel porosity and temperature. The uncertainty in thermal conductivity, estimated from the data of references 4 and 5, is ±20 percent.

(5) Orifice Mismatch. Uncertainty in coolant flow results from inability to size orifices to compensate accurately for fabrication discrepancies in flow passages and in the distribution headers. Tolerances suggest a value of ±5 percent.

(6) Flow Annulus Width. Here again the uncertainty is based on fabrication tolerances. These allow a variation of ±0.0121 centimeter (±0.00475 in.) about the design value of 0.10 centimeter (0.040 in.).

(7) Heat Transfer Coefficient. Reference 6 shows data for liquid metal heat transfer at Peclet numbers below critical with Nusselt numbers varying from 4.9 to 7.8. This is roughly a range of ±25 percent about an average. On this basis, we assume an uncertainty of ±25 percent.

(8) Fuel-Clad Gap Width. Tolerances in fuel outer radius, tungsten liner thickness, and clad inner radius give gap widths at temperature varying from 0.0013 centimeter (0.0005 in.) to 0.0063 centimeter (0.0025 in.). The uncertainty is therefore ±0.0025 centimeter (±0.0010 in.).

(9) Fuel Shift in the Clad. This could occur as a result of fuel rod bowing, and cause the fuel-clad gap width to be non-uniform. The amount of non-uniformity can be stated in terms of local increase or decrease in width. The value of ±0.0038 centimeter (±0.0015 in.) was chosen which is the maximum gap within tolerances for the hot fuel assembly. The uncertainty amounts to ±100 percent of the gap and includes all possibilities except, perhaps, such effects as out of round distortion.

(10) Clad Shifts in Flow Annulus. This depends on fabrication tolerance and also on warpage due to differential expansion resulting from circumferential temperature variations in fuel pin and annulus wall. The distortion is limited by the dimples in the wall. Uncertainty is based on the variation within this limited range. In terms of maximum local increase or decrease in annulus width, this amounts to ±0.020 centimeter (±0.008 in.).

SUMMARY OF ANALYTICAL TECHNIQUE

The maximum fuel pin temperature in the reactor occurs at the centerline of the center pin slightly above core midplane. Its nominal value T^*_{fm} is a function of all the variables that affect it directly when they have their nominal values.

$$T^*_{fm} = T(x^*_1, x^*_2, x^*_3, x^*_4, x^*_5, \ldots, x^*_n) \tag{1}$$

The maximum fuel temperature can be expressed as follows:

$$T_{fm} = T(x_1^* + \Delta x_1, x_2^* + \Delta x_2, x_3^* + \Delta x_3, \quad \cdot \cdot \cdot, x_n^* + \Delta x_n) \qquad (2)$$

If all the increments are taken at such extreme values as to produce maximum temperature, the result corresponds to the general treatment of the deterministic approach or worst case analysis. As an analytical simplification, the temperature deviation or increment $T_{fm} - T_{fm}^*$ can be approximated as the sum of the linear terms of the Taylor series expansion (ref. 2).

$$T_{fm} - T_{fm}^* = \frac{\partial T_{fm}}{\partial x_1} \Delta x_1 + \frac{\partial T_{fm}}{\partial x_2} \Delta x_2 + \frac{\partial T_{fm}}{\partial x_3} \Delta x_3 + \cdot \cdot \cdot + \frac{\partial T_{fm}}{\partial x_n} \Delta x_n \qquad (3)$$

Two important assumptions are implied in the use of the above equation. One is that the variables $x_1, x_2, \ldots, x_n$ are mathematically independent, that is, there are no functional relations among the variables. In addition, the relation between the maximum fuel pin temperature and the other variables must be linear or nearly so in the range in which the equation is applied.

The older technique (ref. 1) for calculating $T_{fm} - T_{fm}^*$ had been the simple multiplication of the individual hot spot factors with the Δx_i's at their maximum possible value. This technique is called the deterministic method. These factors, described in reference 1, can be expressed as follows:

$$f_i = 1 + \frac{\text{change in } T_{fm} \text{ due to uncertainty "i"}}{T_{fm}^* - T_{\text{coolant inlet}}} \qquad (4)$$

The numerator of the fraction in equation (4) can be calculated from equation (3) if the variable has a nearly linear effect on temperature and if the coefficients $\partial T_{fm}/\partial x_i$ are known. Otherwise, the temperature change can be calculated from an expression relating the maximum fuel pin temperature to a variable with every other variable at its nominal value. In either event, a conservative result is likely to be obtained when the product of individual hot spot factors, namely, $\prod_{i=1}^{n} f_i$ is calculated for the overall hot spot factor. In fact, if the input variables are independent and the temperature is linear with respect to the input variables, then the "deterministic" method yields the same maximum temperature as a worst case analysis.

In contrast with the deterministic approach, a technique has recently evolved (ref. 7) that applies some statistical theory of random variables

to the input uncertainties and their effect on the maximum fuel pin temperature. If equation (3) is used with its attendant assumptions then the standard deviation of the maximum fuel pin temperature σ_T can be expressed in terms of the standard deviations, σ_i's of the input variables. Thus

$$\sigma_T^2 = \left(\frac{\partial T_{fm}}{\partial x_1}\right)^2 \sigma_1^2 + \left(\frac{\partial T_{fm}}{\partial x_2}\right)^2 \sigma_2^2 + \left(\frac{\partial T_{fm}}{\partial x_3}\right)^2 \sigma_3^2 + \cdot\cdot\cdot \left(\frac{\partial T_{fm}}{\partial x_n}\right)^2 \sigma_n^2 \qquad \text{(ref. 2)} \tag{5}$$

Hence, if the standard deviations of the input variables can be estimated, then it is possible to estimate the standard deviation of the maximum fuel pin temperature. No assumptions have been made at this point as to the frequency distribution of the input variables (x_i's).

Once the standard deviation of the maximum fuel pin temperature has been calculated, it is possible to estimate the probability of exceeding some temperature limit only by making some assumptions about the frequency distribution of the fuel pin temperature.

At this point, most of the literature makes one or the other of the following assumptions (in addition to those underlying equation (5)): (1) the input variables each have a normal frequency distribution; hence the output variable T_{fm} also has a normal distribution, or (2) the Central Limit Theorem applies as an approximation to the author's case. The Central Limit Theorem states that if equation (3) is applicable then the distribution of T_{fm} will approach the normal distribution as n approaches infinity independently of the distributions of the x_i's. The validity of the second assumption depends on the number of variables that contribute to the major portion of the variance of T_{fm}. Thus, Abernathy (ref. 2) shows that with only five independent variables whose variances contribute $97\frac{1}{2}$ percent of the output variance and are within a factor of 8 of each other, the output variable "closely approximates" a normal distribution, even though the input variables had rectangular distributions. However, as will be discussed later, the approximation is not necessarily close for small or "tail" probabilities.

Finding the distribution of the output variable may also be accomplished directly by applying the method described by Abernathy (ref. 2). This would use the "guessed at" frequency distributions of the input variables. In general, the method would be tedious and the result, at this preliminary stage of design, of such questionable application, that such an approach would not be worth the effort.

Another alternative which yields a very conservative limit as the probability function of the output variable based solely on the standard

deviation calculated from equation (5) is the application of the Chebyshev inequality (ref. 7). Use of Chebyshev's inequality requires only that the variable have a mean and standard deviation. The inequality can be stated in the following way as applied here: The probability that the maximum fuel pin temperature will exceed $\overline{T}_{fm} + n\sigma_T$ is less than $1/n^2$ where n is the number of standard deviations, $\overline{T}_{fm}$ is the population mean of the distribution, and σ_T is the value of the standard deviation. Thus, if T_L is the fuel temperature limit then the probability of exceeding T_L is $\leq 1/((T_L - \overline{T}_{fm})/\sigma_T)^2$. Assuming that $\overline{T}_{fm}$ and σ_T are known exactly, Chebyshev's inequality is the most conservative approach that could be taken at this point since it assumes nothing with respect to the frequency function of the temperature distribution except the existence of a mean and variance. In view of the preliminary nature of the design, it was considered that an estimate of reliability should be made based on the Chebyshev inequality. In addition, the probability, based on the assumption that the frequency distribution of the maximum fuel pin temperature is a normal one, will be calculated. This is the usual procedure from the literature and is certainly much more optimistic than the Chebyshev inequality probability. It has some validity in the fact that the Central Limit Theorem does provide that a linear combination of a large number of independent random variables does tend toward a normal distribution. Once the frequency distributions are more precisely defined then it would be possible (in theory) to completely define the distribution of the maximum fuel pin temperature by a technique such as the one described by Abernathy (ref. 2).

THE ASSUMPTIONS OF INDEPENDENCE AND LINEARITY

Essential to the development of equation (5) are the assumptions of independence among the input variables and linearity between the output and the input variables.

Of the ten input variables considered in this analysis (see table II), the only question as to any dependency arises with respect to those variables which affect the coolant flow rate and the heat transfer coefficient. Variables x_5 (orifice mismatch), x_6 (flow annulus width), and x_{10} (clad shift in flow annulus) all affect coolant velocity which in turn can alter the coolant heat transfer coefficient. However, the critical Peclet number for lithium is 300 (ref. 6) and the Peclet number for the coolant in the flow annulus is about 70. This means that the heat is transferred from the fuel to the coolant by molecular conduction and is essentially independent of the fluid velocity. Hence, the heat transfer coefficient is independent of those variables affecting fluid velocity. However, it was found that the clad shift in the flow annulus does cause a change in the average heat transfer coefficient making the latter a dependent variable. But the influence of the uncertainties in the heat transfer coefficient on the maximum fuel pin temperature is so small that the effect of any interdependence on the standard deviation estimate or the worst case analysis can be

neglected. There is no dependence that is in evidence among any of the other factors.

The linearity of the relation between the fuel temperature and the input variables is assumed without proof. A more detailed investigation of the degree of non-linearity and the error introduced by any non-linearity is left for a future investigation. Some discussions of Taylor expansions to be used in the presence of dependencies and non-linearities are given in references 7 and 8.

CALCULATIONAL PROCEDURE

The calculation is made for the reactor centerline fuel pin except where otherwise stated. Heat generation and temperature in this pin are circumferentially uniform if fuel thickness and density are uniform, if the fuel remains centered in the clad, and if the clad remains centered in the coolant passage. For this ideal situation, since we choose to make the hot spot analysis at the plane where the fuel temperature is a maximum, a radial distribution in the pin is all that is necessary and reference conditions are found with a one-dimensional calculation.

However, the fuel is not, in general, apt to be centered in the clad, nor the clad in the coolant passage. Hence, a two-dimensional calculation is required. This has been done using the steady state heat transfer program STHTP. The program uses a rectangular nodal scheme for handling one, two, or three dimensional problems with heat generation, constant or temperature dependent thermal conductivity, constant contact or film coefficients, and radiative heat transfer with constant emissivity. Cylindrical nodes can be handled with sufficient accuracy if radius ratios are not too large.

Node arrangement for the calculations are shown in figure 3 for the reference case where fuel is centered in the clad and clad is centered in the coolant passage. Temperature drop across the liner was included in the calculation but was insignificant.
Figure 4 shows the arrangement for the calculations with asymmetry in the gas gap and in the coolant passage.

The hot spot calculation is confined to finding a hot spot factor at the beginning of core life assuming 100 percent helium in the gap between the fuel and the clad. Even though fission products escaping into the helium gap during irradiation decrease the thermal conductivity of the gas during operation, the growth of the fuel decreases the gap. As a result, the maximum fuel temperature at the beginning of core life will be greater than any other time during operation.

The procedure for finding the effect on temperature of the various independent factors is, first to make temperature calculations using a reference set of conditions. This establishes T^*_{fm} of equation (1) where

the reference conditions are the nominal values of the variables. Then values of expected maximum variation for each factor affecting temperature are established. The method and values were discussed earlier in the section Discussion of Uncertainties. These are the Δx's of equations (2) and (3). New temperatures are then calculated using the maximum perturbed value of the factor or variable. A ΔT, the difference in temperature between the reference case and perturbed case, is then calculated.

Even though we have established what were termed maximum variations for each factor, we must establish a degree of confidence that the actual value will fall within those maxima. This degree of confidence is related to the standard deviation for a particular distribution. To each assumed maximum variation, a particular number of standard deviations is assigned. The less confidence there was in the maximum variation, the fewer the number of standard deviations were assigned to it, which therefore produced relatively larger estimates of the standard deviations.

The particular number of standard deviations assigned to the expected maximum variation of each variable was discussed in the section on uncertainties. For those variables that were considered to have approximately rectangular distributions, the number of standard deviations is $H/\sigma = \sqrt{3}$ where H is the half range of the variable (ref. 1). These include the variables concerned with dimensional tolerances. Estimation of the standard deviation of the remaining variables are derived primarily from the literature. There was an element of uncertainity as to the applicability of the literature estimates to our own case. Where this element was strong, the degree of confidence in the value was reduced by decreasing the number of standard deviations quoted.

The temperature of the fuel pin is calculated for each factor at its perturbed value. Each difference between nominal and perturbed temperatures $(T_{fmi} - T^*_{fm})$ is then divided by the number of standard deviations estimated for that factor. This means all the temperature differences now correspond to one standard deviation in each factor. This ΔT_i is then the term $(\partial T_{fm}/\partial x_i)\Delta x_i$ in equation (3), with the Δx_i's equal to one standard deviation. Also when squared, it becomes the term $(\partial T_f/\partial x_i)^2\sigma_i^2$ in equation (5) so that the sum of all such squared terms is the variance of the maximum fuel pin temperature. This means equation (5) can be rewritten as follows when a linear relation exists between input and output variable:

$$\sigma_T^2 = \Delta T_1^2 + \Delta T_3^2 + \cdot \cdot \cdot \Delta T_n^2 \qquad (6)$$

where the ΔT's are the temperature rises accompanying a one standard deviation perturbation of the input variables.

The hot spot factors and probabilities are to be calculated based on assumptions concerning the frequency distribution of the maximum fuel pin temperature. The usual technique is to assume that the distribution is

a normal one and that at three standard deviations, the probability of exceeding that temperature (0.0013) is sufficiently small as to call it highly unlikely. It is with this three standard deviation temperature that the hot spot factor is calculated.

As described earlier, the "deterministic" method was used as a worst case analysis approximation. The hot spot factor resulting from this technique is also quoted. The hot spot calculations using both the statistical method and deterministic method are given in Appendix C.

Apart from the hot spot factors, a calculation was made for the probability of exceeding the maximum fuel temperature limit of 1622 K (2460° F). This limit is based on a preliminary estimate established to prevent possible nitrogen embrittlement of the T-111 clad. This limit was established in connection with the work described in reference 9.

CALCULATIONAL INPUT

Data required for making the calculation included information on node connections and dimensions, as stated in the preceeding section. Also included are thermal conductivities, either constant or functions of temperature, emissivities for thermal radiating surfaces, convective heat transfer coefficients, heat generation rate, and sink temperatures. Since the calculation is made on a pin cross-section at one core location, the coolant temperature at that location is taken as the sink temperature. Volumetric heating rate is assumed to be independent of fuel radius and no heat is generated elsewhere. Convective heat transfer is based on Dwyer's correlation (ref. 10).

Lithium properties are those reported by Davison (ref. 11). Properties of the T-111 cladding were obtained from reference 12. UN properties are from references 4 and 5 and helium properties from reference 13.

RESULTS

The values of the perturbed temperature differences that correspond to one standard deviation of the contributing factor or variable are summarized in table II. The components of the variance or $(\Delta T_i)^2$ terms of equation (6) are also listed. The magnitude of one standard deviation of the maximum fuel pin temperature is 31.0 K (55.8° F). Applying the recent conventions of using three standard deviations in the calculation of the hot spot factor the result is a fuel hot spot factor of 1.46. The corresponding clad hot spot factor is 1.59 as shown in table III.

The "deterministic" method of multiplying individual hot spot factors was also used. The extreme unfavorable value of each variable had to be selected. Since a "deterministic" analysis should be a fully

pessimistic approach as far as the coincidence of unfortunate events is concerned, the additional pessimism of assuming each variable's maximum value to be at three standard deviations was used. This was done even though the value on a tolerance limited variable was well outside its physical possibility. This, however, could compensate for gross errors in inspection or some relaxation of tolerances as the design and fabrication proceeds. The individual hot spot factors are listed in table V. The cumulative hot spot factor is 2.18 so that the maximum fuel pin temperature under "deterministic" (approximate worst case) conditions would be 1610 K (2438° F).

In the design study, a conservative fuel temperature limit of 1622 K (2460° F) has been established in order to avoid the possibility of embrittlement of the clad due to the formation of nitrides from the nitrogen vapor in the fuel. This means that the "deterministic" analysis indicates only a small safety margin of 12 K (21.6° F) between the maximum fuel pin temperature and the fuel temperature limit. This situation warrants a closer look at a more realistic approach inherent in a statistical hot spot factor analysis. One such method gives a maximum fuel pin temperature of 1463 K using the assumptions that the frequency distribution is normal and that the probability of exceeding three standard deviations is essentially zero (for exact normality and exactly known mean and variance it is 0.13 percent).

The reactor will thus be safe with respect to the fuel temperature limit of 1622 K by this analysis. This limit stands 8.1 standard deviations away from the nominal fuel pin temperature. However, as illustrated on page 235 of reference 7, while the normal distribution is often a good approximation to a real situation in the central range of a distribution (say out to two standard deviations) it becomes increasingly in error at the tails (namely beyond two standard deviations). The assumption of normality can be avoided using the Chebyshev (or Tchebychev) inequality.[1] Using it, the probability is 0.015 that the central fuel pin will exceed the fuel temperature limit of 1622 K.

This represents a small probability that the central fuel pin will exceed the fuel temperature limit. However, there are 247 pins in the reactor, albeit, at lower temperatures. Let P_{Fi} be the probability of failure or the probability that the fuel temperature limit will be exceeded by the i^{th} pin of the N or 247 pins. The success probability for each pin $P_{si} = (1 - P_{Fi})$ was calculated based on its maximum operating temperature.

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[1]In using the Chebyshev inequality, it is necessary to assume that the nominal or design temperature T^*_{fm} coincides with the mean of the temperature distribution $\overline{T}_{fm}$. This is a reasonable assumption in view of the pessimism implied in the use of the Chebyshev inequality and that highly skewed distributions are not expected for any of the variables.

The probability that all the pins will operate below the limit is the product $\prod_{1}^{N} P_{si}$. This assumes the events are independent of each other from pin to pin. Most of the variables are local with respect to each fuel pin. Only two of the variables, namely, overall power and heat transfer coefficient are global with respect to all the fuel pins. Furthermore, the fuel clad gap width greatly overshadows all the other variables in importance in contributing to fuel pin temperature variability (see table III). In view of these considerations, the 247 different fuel pins will be assumed to survive or fail, statistically independently of each other. The probability that at least one pin fail is simply $1 - \prod_{1}^{N} P_{si}$.

The maximum nominal operating temperature was calculated for each pin and the standard deviation scaled to that for the hottest pin in proportion to the amount of heat generated. With these numbers, the Chebyshev inequality was used to calculate the failure probability P_{Fi} for each pin. The probability of failure of at least one pin at the core could be as much as 0.82. In other words, until the standard deviations of the input variables can be better defined and reduced and/or the frequency distribution of these variables or the output variable (fuel temperature) are known or can be assumed with more confidence, fuel pin integrity with respect to the failure of the clad by nitrogen embrittlement should remain somewhat in doubt. It should be emphasized that pessimism upon pessimism has been heaped on this calculation as far as expanded standard deviations, worst possible frequency distributions, and the fuel pin temperature limit itself is concerned. As indicated previously, even the "deterministic" analysis only shows a marginal chance of the central pin failing by this mode, hence, less chance by any other pin.

Another point that should be made is that while the overall power adjustment is the second greatest contributor to the uncertainty in the fuel pin temperature, it is only a transient effect and would probably not have any great influence on failure by nitrogen embrittlement of the clad which is a long term effect. However, elimination of this uncertainty reduces the standard deviation by only 1 K.

The calculations showed that the fuel-clad gap is the most significant factor in determining the fuel hot spot factor. In this respect, the dimension should be kept strictly within tolerance. The uncertainty of this factor represents 28.2 K (50.8° F) of the total calculated standard deviation of 31.0 K (55.8° F)(see table II). The frequency distribution of the fuel-clad gap will be the prime influence on the distribution of the fuel temperature. This means that since the frequency distribution of the gap can probably be described as somewhat rectangular in shape, use of the Chebyshev inequality, where no assumption is made as to the

distribution, or in a sense assumes the worst possible distribution, is too pessimistic even at this point.

The variables, local fuel thickness, orifice mismatch, heat transfer coefficient, and flow annulus space have little effect on the maximum fuel temperature. Also, any shifting of the fuel in the clad or shifting of the clad in the flow channel would cause a much smaller perturbation in maximum fuel temperature (less than 2 K (3^{o} F) for one standard deviation).

CONCLUDING REMARKS

Calculations have been made to find temperature distributions in some of the hottest fuel pins in a reactor core, and to estimate the probability of extreme temperatures to be encountered in the fuel and in the clad. Hot spot results are preliminary because of the preliminary nature of the design information used, the paucity of material property data, and the lack of hardware for use in estimating condition and behavior of the parts involved. The following results should, however, be useful in the continuing design work:

1. The nominal maximum fuel temperature occurs in the centerline fuel pin and has a value of 1370 K (2006^{o} F).

2. Temperature decreases radially, the maximum nominal values in Zones II for fuel being 1353 K (1982^{o} F) and in Zone III is 1333 K (1950^{o} F).

3. A "deterministic" analysis of the fuel pin temperature yields a maximum temperature of 1610 K (2438^{o} F). This indicates only a small margin of safety below the 1622 K (2460^{o} F) which it is estimated would cause failure due to nitrogen embrittlement of the clad.

4. A recently favored statistical approach gives a maximum fuel pin temperature of 1463 K (2173^{o} F). The technique assumes a normal frequency distribution of the fuel temperature and sets three standard deviations above the mean or nominal value as the upper fuel temperature that can be expected. The standard deviation in fuel temperature by this technique is 31.0 K (55.8^{o} F). The fuel pin hot spot factor is 1.46.

5. The probability of exceeding a fuel temperature limit of 1622 K (2460^{o} F) in the central fuel pin could be as much as 0.015. This statement is based on the Chebyshev inequality and assumes only that the mean and standard deviation of the fuel temperature are known. Thus, there is no assumption as to the distribution as was made in the calculation of the maximum expected fuel temperature in 4 above. The method is extremely pessimistic, but does show the need for a reasonable grasp on the frequency distributions of the variables. For if we extend the method to all

247 fuel pins, the probability for the failure of one or more of them by exceeding the limit could be as much as 0.82 which is much too high and could occur if the fuel temperture has a very unfavorable frequency distribution.

6. In this light, the probability distribution function of the fuel-clad gap is the most significant contributor to the standard deviation of the fuel pin temperature. Hence, the statistical nature of this variable should be closely studied to insure the integrity of the fuel pins. Even at this point, it is possible to say that the frequency distribution of this variable will be somewhat rectangular in shape making probabilities based on the Chebyshev inequality too pessimistic.

APPENDIX A

SYMBOLS

A	cross sectional area
$c_1,c_2,c_3,$ c_4,c_5	arbitrary constants
F	fuel temperature rise
f	hot spot factor
H	half range of symmetrically distributed variable
k	thermal conductivity
n	number of standard deviations
P_F	probability of failure
P_s	probability of success
Q	heat generation rate per unit volume
q	heat transfer per unit area
r	radius
T	temperature
T_L	fuel temperature limit
ΔT	temperature difference
V_f	ratio of fuel volume to cell volume
w	fuel-clad gap width
x	variable which contributes to fuel temperature uncertainties
Δx	change in variable from its mean value
$\delta_{m;i,j}$	temperature rise from location j to location $j+1$ when perturbed by off-design condition of i^{th} variable
$\delta_{m,j}$	temperature rise from location j to location $j+1$
σ	standard deviation
σ_T	standard deviation of dependent variable

Subscripts:

cl clad

f fuel

i i^{th} variable $(i = 1, 2, 3, \ldots, 10)$

in inner

j location along path of temperature rise $(j = 1, 2, \ldots, 6)$

m value of variable at the position where it achieves its maximum value

n n^{th} or last variable

ref reference case

Superscripts:

* nominal or design value

— denotes mean value of variable

APPENDIX B

CORE DESIGN RADIAL AND AXIAL TEMPERATURE DISTRIBUTION

The axial power distribution is assumed to be the same at all radii in the core. It has a chopped cosine form as shown in table IV and figure 5, with an axial peak to average power ratio of 1.23. The radial peak to average power ratio is 1.33. That is, the highest power Zone I fuel pin, the centerline pin generates 1.33 times the power of an average pin in the core. For the Zone II pin generating the highest power, this ratio is 1.20 and for Zone III, it is 1.01. Using these power distributions and the data in table I, the curves in figure 5 were obtained showing maximum temperature in a cross-section of the fuel as a function of axial location, assuming a gap between fuel and clad that is uniform around the pin and a coolant flow passage that is uniform around the pin. These indicate that the highest fuel temperature in the core, under normal operating conditions, is in the centerline fuel pin and slightly downstream of the midpoint of this pin. This temperature is normally 1373 K (2006° F). The fuel temperature drops off both axially and radially from these values. The maximum fuel temperature in Zone II is 1353 K (1982° F) and in Zone III, 1333 K (1950° F). The clad temperatures at the location of maximum fuel temperatures are 1231 K (1756° F), 1225 K (1745° F), and 1215 K (1728° F) for Zones I, II, and III, respectively.

The clad temperature reaches its maximum in each zone at the coolant outlet. However, for the calculation of hot spot factors, for both the fuel and clad, temperatures at the fuel maximum point were used. It makes computations simpler to calculate hot spot factors at the same point for the fuel and the clad. It is a valid approach also in the sense that hot spot factors for the clad will be conservative at this location, that is, greater than calculated at the clad maximum temperature location.

All the calculations omit any consideration of axial heat transfer. We may assume that this has very little effect on results, because a calculation shows that in the fuel in the region of highest axial temperature gradient where there is a heat generation of about 5.9 kW (20 000 Btu/hr) to be disposed of, less than 3.5 W (12 Btu/hr) is transferred axially out of the fuel.

APPENDIX C

HOT SPOT CALCULATIONS

Variability of Temperature Rises

Table V shows details of the calculation of the composite standard deviation for clad temperature and for fuel temperature, using temperatures obtained from one dimensional and two dimensional heat transfer calculations at the axial position of maximum fuel temperature of the centerline fuel pin. Temperature rise from the inlet coolant temperature to the fuel hot spot was divided into five increments as shown below.

1. Coolant temperature rise

2. Film temperature rise from coolant to clad outer surface

3. Clad temperature rise from outer surface to inner surface

4. Temperature rise across the clearance gap between clad and fuel

5. Temperature rise within the fuel

Some of the factors contributing uncertainty to the temperatures had their effect on all increments, others on only one or two increments. Factors affecting the first three items above were used in finding the standard deviation for the clad temperature. All 10 factors contributed to the standard deviation for the fuel temperature.

The contribution of each factor (the column headings of table V) to the variability of each of the five temperature rise increments was calculated as summarized by table V. The calculations used repeated applications of equation (3) for each of the five temperature rise increments. These five increments exist between locations that can be defined as follows.

Location	Subscript, j
Annulus inlet	1
Local coolant	2
Clad outer surface	3
Clad inner surface	4
Fuel outer surface	5
Fuel hot spot	6

The five increments of temperature can also be identified with the subscript j as follows:

Increment	Subscript, j
Coolant rise	1
Film rise	2
Clad rise	3
Gap rise	4
Fuel rise	5

The nominal temperature at any location can be defined as $T^*_{m,j}$ and the increment of nominal temperature can be defined as $\delta^*_{m,j}$ and thus

$$\delta^*_{m,j} = T^*_{m,j+1} - T^*_{m,j} \tag{C1}$$

Thus for each of the subtables of table V, the third line of parts (a), (b), (c), (d), and (e) gives the quantity $\delta^*_{m,1}$, $\delta^*_{m,2}$, $\delta^*_{m,3}$, $\delta^*_{m,4}$, and $\delta^*_{m,5}$, respectively.

The next step is to calculate increments of temperature rise $\delta_{m;i,j}$ as a function of each independent variable or factor x_i and the uncertainty, Δx_i, in that variable, from the appropriate functional relation:

$$\delta_{m;i,j} = T_{m,j+1}(x_i + \Delta x_i) - T_{m,j}(x_i + \Delta x_i)$$

The difference between $\delta_{m;i,j}$ and $\delta^*_{m,j}$ is the amount by which the j^{th} rise is changed due to a change in the i^{th} factor by the amount of its uncertainty Δx_i.

The number of standard deviations of x_i associated with Δx_i is assumed (as previously discussed) to be n_i (as given in table V) and the resulting estimate of the standard deviation, σ_i, of the variable x_i is

$$\sigma_i = \Delta x_i / n_i \tag{C2}$$

Correspondingly, the difference between $\delta_{m;i,j}$ and $\delta^*_{m,j}$ (which had been computed from Δx_i) can therefore be reduced to a value attributable to just one standard deviation of x_i by dividing the difference by n_i. The result is defined as $\Delta T_{m;i,j}$:

$$\Delta T_{m;i,j} = (\delta_{m;i,j} - \delta^*_{m,j})/n_i \tag{C3}$$

Equation (C3) was used to compute the sixth line of each of tables V(a) through V(e).

Combination of Variances

The independent variables were listed as the column headings of table V. They are assumed to be statistically independent as one of the basic assumptions justifying the use of equation (6). This statistical independence is the basis of equation (5) which was specialized to equation (6) and is the reason why the increments of equation (6) are added together as a sum of their squares. By way of contrast, the temperature increments given in the sixth line of each of tables V(a) to (e) for any one column are not statistically independent of each other, but are each tied functionally to the single random variable given by the column heading. Because of this nonindependence, their effects are directly additive in producing the net temperature effect on the clad or on the fuel. Thus the increment of clad temperature variation associated with one standard deviation of x_i is the direct sum of the increments given by each of the sixth lines (eq. (C3)) of tables V(a) to (c):

$$\Delta T_{c\ell,i} = \sum_{j=1}^{3} \Delta T_{m;i,j} \tag{C4}$$

Results of this summation are given in the first two columns of table III.

Similarly, the columns headed $\Delta T_{f,i}$ of table II conform to the same basic discussion as do the columns labeled $\Delta T_{c\ell,i}$ of table III. The $\Delta T_{f,i}$ values are merely those summations of the sixth lines of tables V(a) through (e) which give the increments of fuel hot spot temperature associated with one standard deviation of x_i, as calculated from increments given by equation (C3) according to the direct summation:

$$\Delta T_{f,i} = \sum_{j=1}^{5} \Delta T_{m;i,j} \tag{C5}$$

The squares (tables II and III) of $\Delta T_{f,i}$ and $\Delta T_{c\ell,i}$, respectively, are needed because the next combination of temperature increments will be with respect to variables assumed to be statistically independent. Whereas equations (C4) and (C5) exhibited summations with respect to j of increments that were nonstatistically independent, the combination of those summations with respect to i will be over statistically independent variables and is done in the manner of equation (5) as specialized to equation (6). In particular, equation (6) was specialized to the variance of the clad inner temperature as equation (C6) and to the variance of the fuel hot spot temperature as equation (C7)

$$\sigma^2_{T,c\ell} = \sum_{i=1}^{10} (\Delta T_{c\ell,i})^2 \tag{C6}$$

$$\sigma_{T,f}^2 = \sum_{i=1}^{10} (\Delta T_{f,i})^2 \tag{C7}$$

The results of the computations with equations (C6) and (C7) on the entries in tables III and II, respectively, are given in the bottom lines of the same tables. The standard deviations (which are the square roots of the variances $\sigma_{T,c\ell}^2$ and $\sigma_{T,f}^2$) are given in the footnotes to tables III and II.

Individual Hot Spot Factors

A definition of an individual hot spot factor was given as equation (4). To perform an approximate worst case or "deterministic" analysis, the numerator of equation (4) is replaced by the i^{th} term of equation (3) with the Δx_i in that term replaced by $3\sigma_i$. Under these assumptions, the numerator of equation (4) is given by

$$\frac{\partial T_{fm}}{\partial x_i}(3\sigma_i) = 3\Delta T_{f,i} \tag{C8}$$

where $\Delta T_{f,i}$ was given in table II. Correspondingly, "individual hot spot factors" were calculated as given in table VI from temperature increments $\Delta T_{f,i}$ given in table II, according to the equation

$$f_i = 1 + \frac{3\Delta T_{f,i}}{(T_{fm} - T_{coolant\ inlet})_{ref}} \tag{C9}$$

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TABLE I. - FUEL PIN HEAT TRANSFER DATA

Core average heat flux at clad surface, W/m^2 $(Btu/(hr)(ft^2))$	390 000 (124 000)
Heat transfer coefficient, clad-to-coolant, $W/(m^2)(K)(Btu/(hr)(ft^2)(^oF))$	173 000 (30 460)
Outside diameter of tube containing coolant, cm (in.)	2.159 (0.850)
Inside diameter of tube containing coolant, cm (in.)	2.108 (0.830)
Outside diameter of clad containing coolant, cm (in.)	1.905 (0.750)
Thickness of clad containing coolant, cm (in.)	0.160 (0.063)
Outside diameter of fuel, cm (in.)	1.577 (0.621)
Inside diameter of fuel, Zone I, cm (in.)	0.762 (0.300)
Inside diameter of fuel, Zone II, cm (in.)	0.678 (0.267)
Inside diameter of fuel, Zone III, cm (in.)	0.480 (0.193)
Width of gap between fuel and clad, cm (in.)	0.0038 (0.0015)
Thermal conductivity of clad, $W/(m)(K)(Btu/(hr)(ft)(^oF)$	66.6 (38.5)
Thermal conductivity of gas, $W/(m)(K)(Btu/(hr)(ft)(^oF)$	0.389 (0.222)
Thermal conductivity of fuel, $W/(m)(K)(Btu/(hr)(ft)(^oF)$	25.6 (14.7)

TABLE II. - FUEL HOT SPOT DATA

Contributing variable	$\Delta T_{f,i}$		$(\Delta T_{f,i})^2$	
	K	$^\circ F$	K^2	$^\circ F^2$
1. Overall power adjustment	8.3	14.9	68.9	223.2
2. Local power	4.2	7.6	17.6	57.2
3. Local fuel thickness	0.6	1.1	0.36	1.17
4. Local thermal conductivity in fuel	6.5	11.7	42.3	136.9
5. Orifice mismatch	1.3	2.3	1.69	5.5
6. Flow annulus width	5.6	10.1	31.4	101.6
7. Heat transfer coefficient	0.45	0.81	0.20	0.66
8. Fuel-clad gap width	28.2	50.8	795.2	2577
9. Fuel shift in clad	2.0	3.6	4.0	12.9
10. Clad shift in flow annulus	1.3	2.3	1.69	5.5
$\sigma^2_{T,f}$			963.3	3121.2

$\sigma_{T,f} = 31.0\ \mathrm{K}\ (55.8^\circ\ \mathrm{F})$

$$\text{Hot spot factor} = 1 + \frac{3\sigma_{T,f}}{(T_{fm} - T_{coolant\ inlet})_{ref}} = 1.46$$

$$\left.\begin{array}{l} T_{fm} \\ \\ T_{coolant\ inlet} \end{array}\right\} \begin{array}{l} \text{ref } 1370\ \mathrm{K}\ (2006^\circ\ \mathrm{F}) \\ \\ \quad 1166.5\ \mathrm{K}\ (1640^\circ\ \mathrm{F}) \end{array}$$

TABLE III. - CLAD HOT SPOT DATA

Contributing parameter	$\Delta T_{c\ell,i}$		$(\Delta T_{c\ell,i})^2$	
	K	^oF	K^2	$^oF^2$
1. Overall power adjustment	2.7	4.9	7.3	23.6
2. Local power	1.4	2.5	2.0	6.4
3. Local fuel thickness	----	-----	-----	------
4. Local thermal conductivity in fuel	----	-----	-----	------
5. Orifice mismatch	1.3	2.3	1.7	5.5
6. Flow annulus width	5.6	10.1	31.4	101.6
7. Heat transfer coefficient	0.45	0.81	0.20	0.66
8. Fuel-clad gap width	----	-----	-----	------
9. Fuel shift in clad	6.4	11.5	41.0	132.7
10. Clad shift in flow annulus	8.9	16.0	79.2	256.6
$\sigma^2_{T,c\ell}$			162.7	527.5

$\sigma_{T,c\ell}$ = 12.8 K (23.0° F)

$$\text{Hot spot factor} = 1 + \frac{3\sigma_{T,c\ell}}{(T_{clm} - T_{coolant\ inlet})_{ref}} = 1.59$$

$$\left.\begin{array}{l} T_{clm} \\ \\ T_{coolant\ inlet} \end{array}\right\} \begin{array}{l} \text{ref } 1231\text{ K } (1760^\circ\text{ F}) \\ \\ \quad 1166.5\text{ K } (1640^\circ\text{ F}) \end{array}$$

TABLE IV. - AXIAL POWER DISTRIBUTION

Relative distance from inlet end	Power relative to average pin power
0.0 (inlet)	0.605
0.1	.815
.2	.990
.3	1.12
.4	1.20
.5	1.23
.6	1.20
.7	1.12
.8	.990
.9	.815
1.0 (outlet)	.605

TABLE V. - CALCULATION OF NOMINAL AND PERTURBED TEMPERATURE RISES.

[Temperatures in kelvin.]

Variable	Overall power adj.	Local power density	Local fuel thickness	Local thermal conductivity in fuel	Orifice mismatch	Flow annulus width	Heat transfer coefficient	Fuel-clad gap width	Fuel shift in clad	Clad shift in flow annulus
No. of std. deviations, n_i	2	2	1.732	2	1.732	1.732	2	1.732	1.732	1.732
Uncertainty, Δx_i	±8.0 %	±4.0 %	±0.0038 cm	±20 %	±5.0 %	±0.0121 cm	±25.0 %	±0.0025 cm	±0.0038 cm	±0.0203 cm
(a) Coolant temperature rise										
Local coolant temp., $T^*_{m,2}$	1212.3	1212.3	--------	--------	1212.3	1212.3	--------	--------	1212.3	1212.3
Annulus inlet temp., $T^*_{m,1}$	1166.5	1166.5	--------	--------	1166.5	1166.5	--------	--------	1166.5	1166.5
$\delta^*_{m,1}$	45.8	45.8	--------	--------	45.8	45.8	--------	--------	45.8	45.8
$\delta_{m;i,1}$	49.5	47.7	--------	--------	48.1	55.5	--------	--------	50.4	62.2
$\delta^*_{m,1} - \delta_{m;i,1}$	3.7	1.9	--------	--------	2.3	9.7	--------	--------	4.6	16.4
$\Delta T_{m;i,1}$	1.9	1.0	--------	--------	1.3	5.6	--------	--------	2.7	9.5
(b) Film temperature rise from coolant to clad outer surface										
Clad outer surface temp., $T^*_{m,3}$	1216.0	1216.0					1216.0		1216.0	1216.0
Local coolant temp., $T^*_{m,2}$	1212.3	1212.3					1212.3		1212.3	1212.3
$\delta^*_{m,2}$	3.7	3.7					3.7		3.7	3.7
$\delta_{m;i,2}$	3.98	3.87					4.6		5.5	2.7
$\delta^*_{m,2} - \delta_{m;i,2}$	0.28	0.17					0.9		1.8	-1.0
$\Delta T_{m;i,2}$	0.14	0.09					0.45		1.0	-0.6

[a] Reference calculation.

[b] Calculated uncertainty.

TABLE V. - Concluded. CALCULATION OF NOMINAL AND PERTURBED TEMPERATURE RISES

[Temperatures in kelvin.]

Variable	Overall power adj.	Local power density	Local fuel thickness	Local thermal conductivity in fuel	Orifice mismatch	Flow annulus width	Heat transfer coefficient	Fuel-clad gap width	Fuel shift in clad	Clad shift in flow annulus
(c) Clad temperature rise from outer surface to inner surface										
Clad inner surface temp., $T^*_{m,4}$	1231.0	1231.0							1231.0	1231.0
Clad outer surface temp., $T^*_{m,3}$	1216.0	1216.0							1216.0	1216.0
$\delta^*_{m,3}$	15.0	15.0							15.0	15.0
$\delta_{m;i,3}$	16.2	15.67							19.7	15.0
$\delta^*_{m,3} - \delta_{m;i,3}$	1.2	0.67							4.7	0.0
$\Delta T_{m;i,3}$	0.6	0.34							2.7	0.0
(d) Temperature rise across the clearance gap between clad and fuel										
Fuel outer surface temp., $T^*_{m,5}$	1305	1305	1305					1305	1305	1305
Clad inner surface temp., $T^*_{m,4}$	1231.0	1231.0	1231.0					1231.0	1231.0	1231.0
$\delta^*_{m,4}$	74.0	74.0	74.0					74.0	74.0	74.0
$\delta_{m;i,4}$	80.1	77.0	74.2					124.0	4.6	70.1
$\delta^*_{m,4} - \delta_{m;i,4}$	6.1	3.0	0.2					50.0	-69.4	-3.9
$\Delta T_{m;i,4}$	3.1	1.5	0.1					28.8	-40.0	-2.2
(e) Temperature rise within fuel										
Hot spot temp., $T^*_{m,6}$	1370	1370	1370	1370				1370	1370	1370
Fuel outer surface temp., $T^*_{m,5}$	1305	1305	1305	1305				1305	1305	1305
$\delta^*_{m,5}$	65	65	65	65				65	65	65
$\delta_{m;i,5}$	70.0	67.5	65.8	77.9				63.9	126.7	56.6
$\delta^*_{m,5} - \delta_{m;i,5}$	5.0	2.5	0.8	12.9				-1.05	61.7	-9.4
$\Delta T_{m;i,5}$	2.5	1.3	0.5	6.5				-0.6	35.6	-5.4

TABLE VI. - "DETERMINISTIC" (APPROX. WORST CASE) TEMPERATURE AND HOT SPOT FACTOR

Contributing variable	Individual hot spot factor, f_i
1. Overall power adjustment	1.122
2. Local power	1.062
3. Local fuel thickness	1.009
4. Local thermal conductivity in fuel	1.096
5. Orifice mismatch	1.019
6. Flow annulus width	1.083
7. Heat transfer coefficient	1.007
8. Fuel-clad gap width	1.416
9. Fuel shift in clad	1.030
10. Clad shift in flow annulus	1.019

$$\prod_{i=1}^{10} f_i = \text{total hot spot factor} = 2.176$$

Maximum fuel temperature = 1610° F

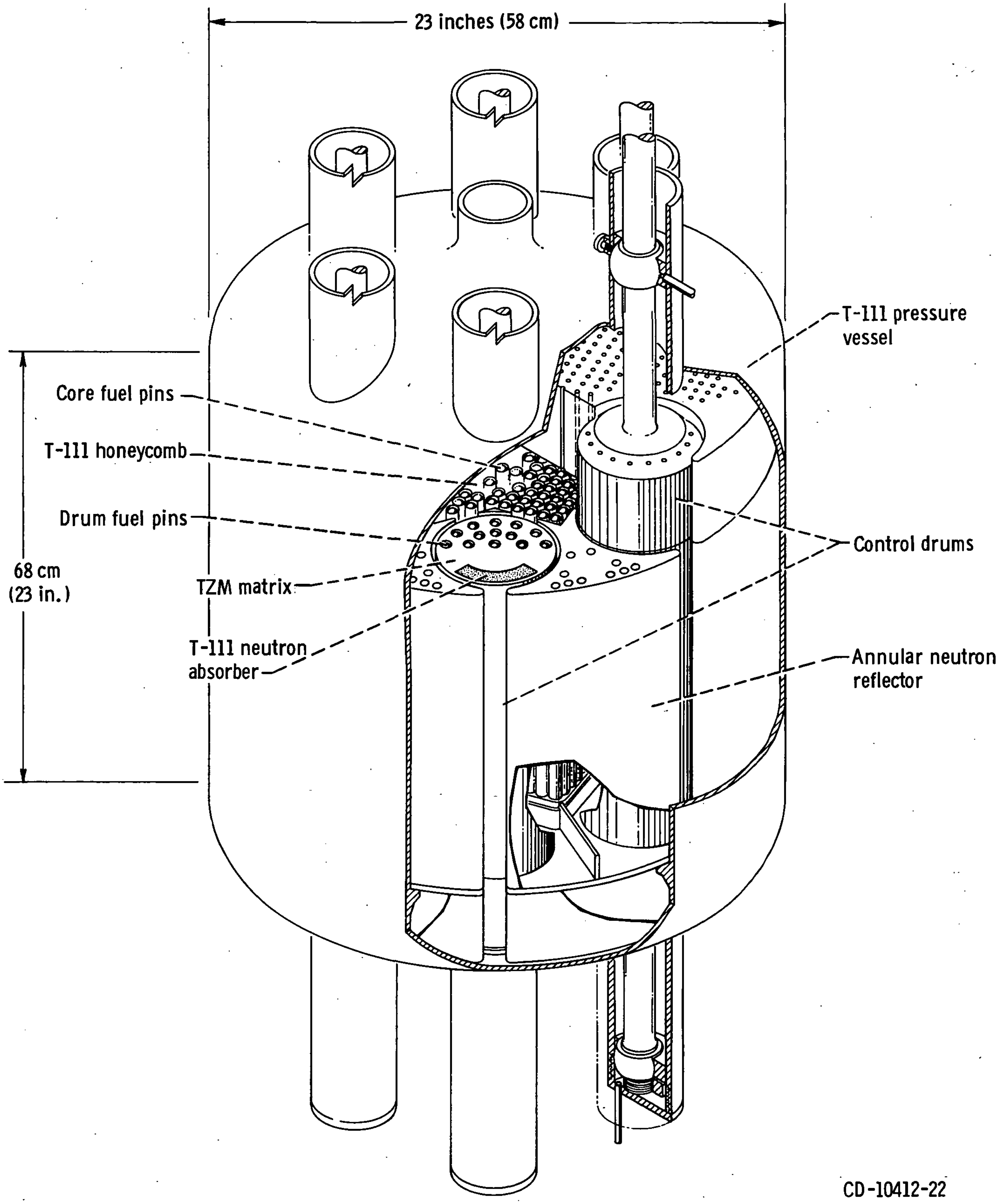

Figure 1. - Nuclear powerplant reactor.

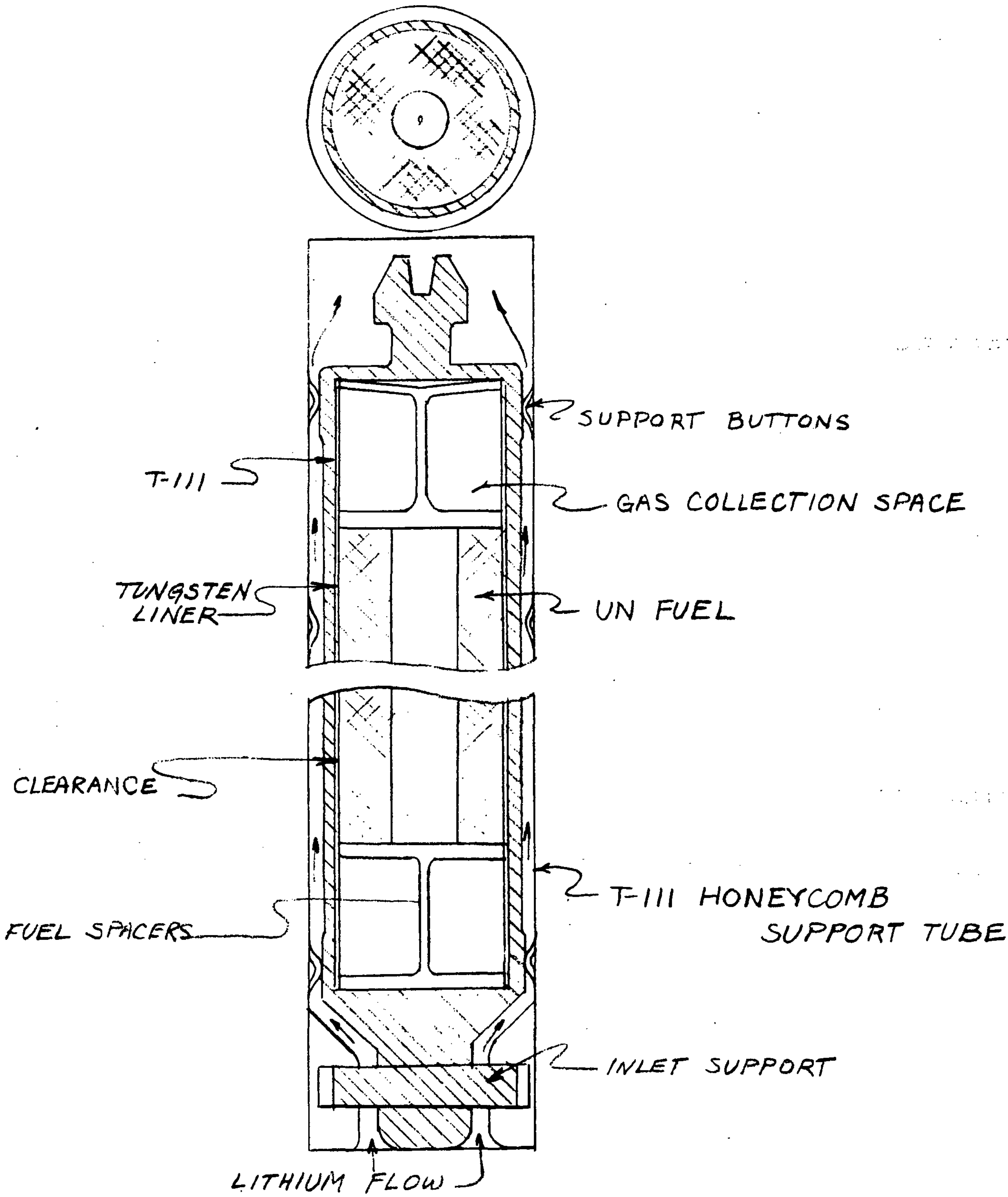

FIG 2 - TYPICAL FUEL PINS

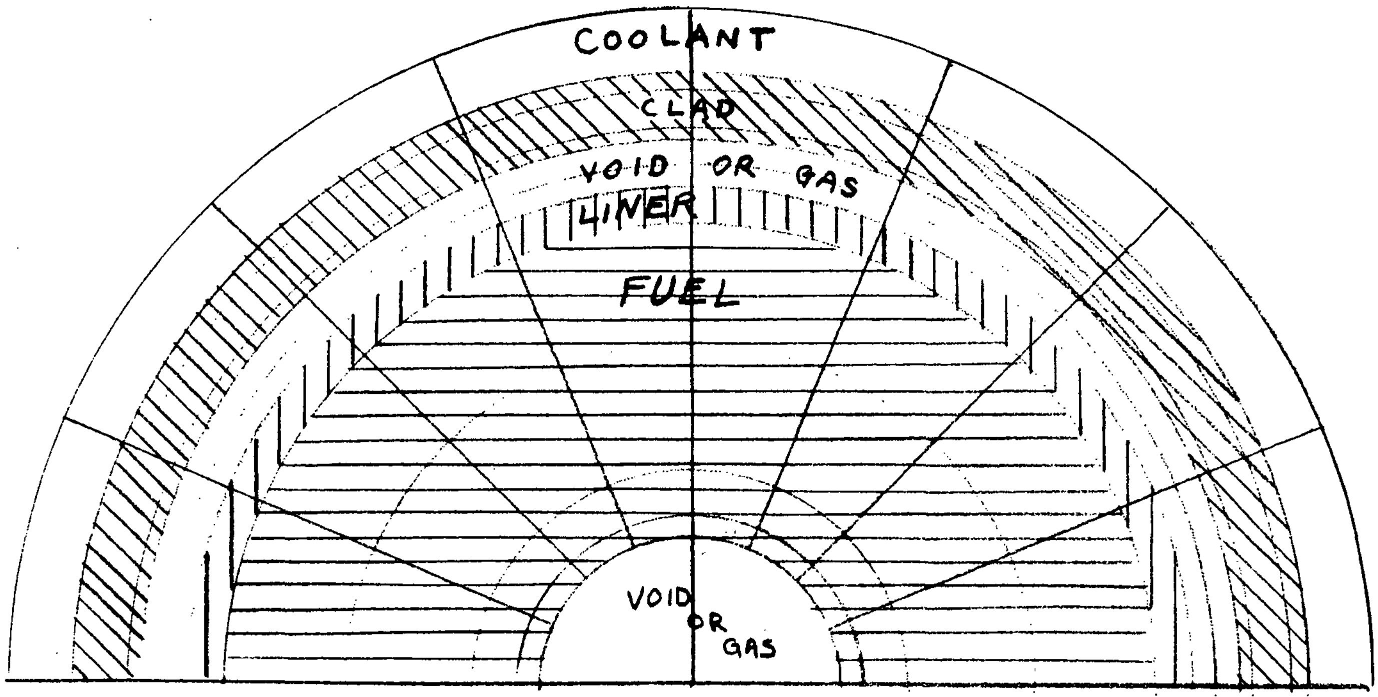

FIGURE 3 - NODE ARRANGEMENT FOR SYMMETRIC CASES

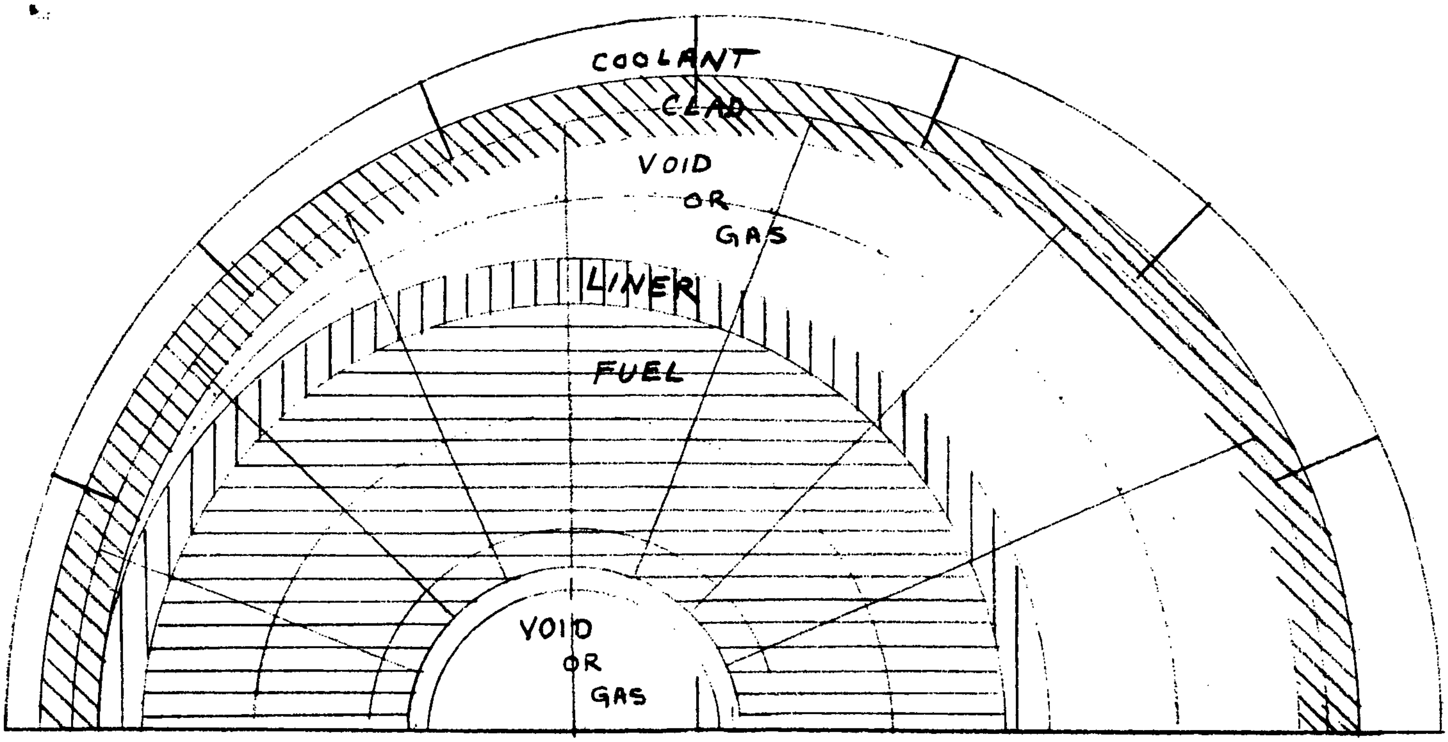

FIGURE 4 NODE ARRANGEMENT FOR ASYMMETRIC CASES

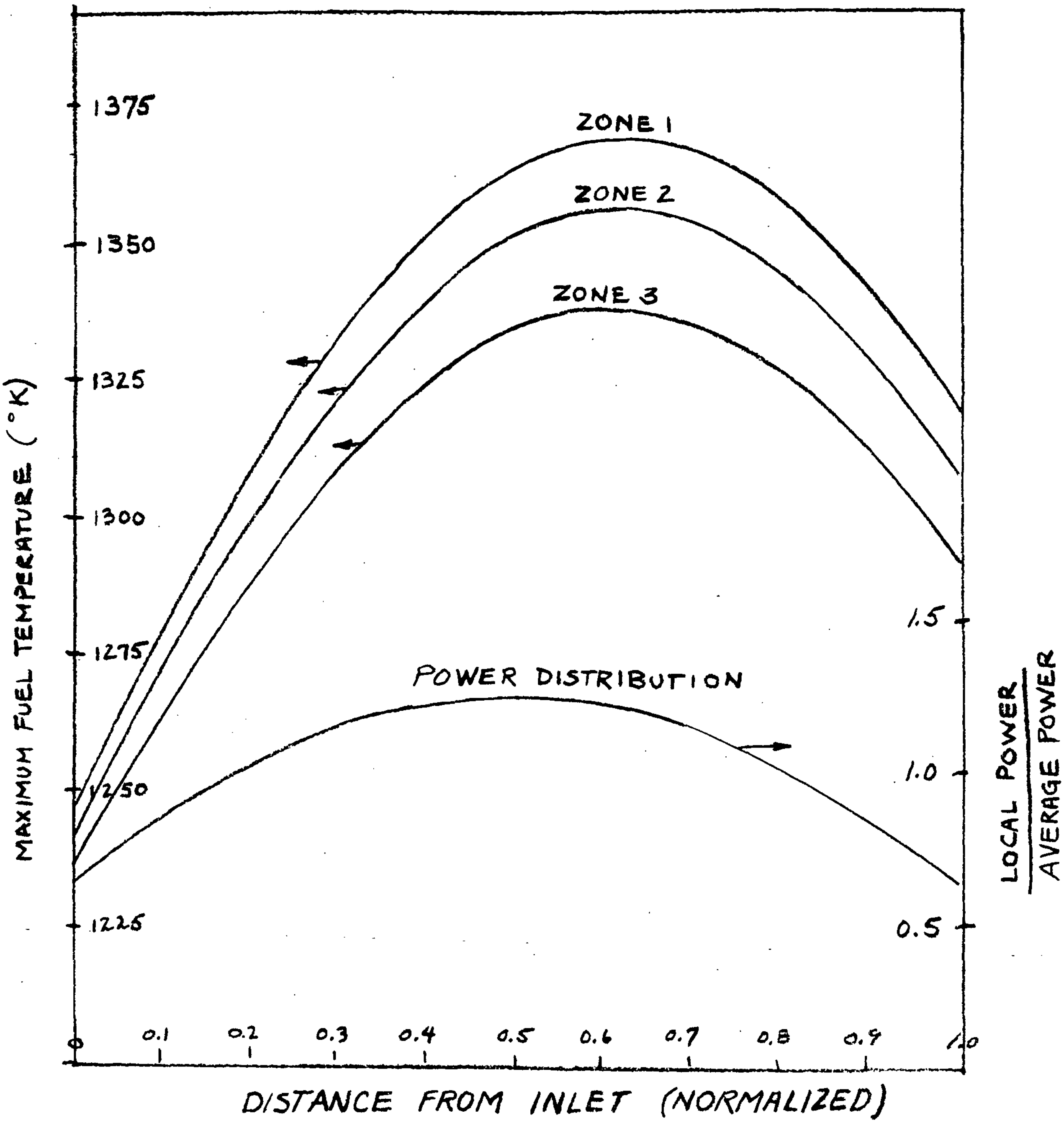

FIGURE 5 NOMINAL TEMPERATURE DISTRIBUTION IN THE FUEL IN THE HIGHEST TEMPERATURE FUEL PIN FOR EACH ZONE